

Pythagorean relations

$$\sin^2 \alpha + \cos^2 \alpha = 1, \quad 1 + \tan^2 \alpha = \sec^2 \alpha, \quad 1 + \cot^2 \alpha = \csc^2 \alpha$$

Angle-sum and angle-difference relations

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\cot(\alpha + \beta) = \frac{\cot \beta \cot \alpha - 1}{\cot \beta + \cot \alpha}$$

$$\cot(\alpha - \beta) = \frac{\cot \beta \cot \alpha + 1}{\cot \beta - \cot \alpha}$$

$$\sin(\alpha + \beta) \sin(\alpha - \beta) = \sin^2 \alpha - \sin^2 \beta = \cos^2 \beta - \cos^2 \alpha$$

$$\cos(\alpha + \beta) \cos(\alpha - \beta) = \cos^2 \alpha - \sin^2 \beta = \cos^2 \beta - \sin^2 \alpha$$

Double-angle relations

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha = \frac{2 \tan \alpha}{1 + \tan^2 \alpha}$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 2 \cos^2 \alpha - 1 = 1 - 2 \sin^2 \alpha = \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}, \quad \cot 2\alpha = \frac{\cot^2 \alpha - 1}{2 \cot \alpha}$$

Multiple-angle relations

$$\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$$

$$\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$$

$$\sin 4\alpha = 4 \sin \alpha \cos \alpha - 8 \sin^3 \alpha \cos \alpha$$

$$\cos 4\alpha = 8 \cos^4 \alpha - 8 \cos^2 \alpha + 1$$

$$\sin 5\alpha = 5 \sin \alpha - 20 \sin^3 \alpha + 16 \sin^5 \alpha$$

$$\cos 5\alpha = 16 \cos^5 \alpha - 20 \cos^3 \alpha + 5 \cos \alpha$$

$$\sin 6\alpha = 32 \cos^5 \alpha \sin \alpha - 32 \cos^3 \alpha \sin \alpha + 6 \cos \alpha \sin \alpha$$

$$\cos 6\alpha = 32 \cos^6 \alpha - 48 \cos^4 \alpha + 18 \cos^2 \alpha - 1$$

$$\sin n\alpha = 2 \sin(n-1)\alpha \cos \alpha - \sin(n-2)\alpha$$

$$\cos n\alpha = 2 \cos(n-1)\alpha \cos \alpha - \cos(n-2)\alpha$$

$$\tan 3\alpha = \frac{3 \tan \alpha - \tan^3 \alpha}{1 - 3 \tan^2 \alpha}$$

$$\tan 4\alpha = \frac{4 \tan \alpha - 4 \tan^3 \alpha}{1 - 6 \tan^2 \alpha + \tan^4 \alpha}$$

$$\tan n\alpha = \frac{\tan(n-1)\alpha + \tan \alpha}{1 - \tan(n-1)\alpha \tan \alpha}$$

Function-product relations

$$\sin \alpha \sin \beta = \frac{1}{2} \cos(\alpha - \beta) - \frac{1}{2} \cos(\alpha + \beta)$$

$$\cos \alpha \cos \beta = \frac{1}{2} \cos(\alpha - \beta) + \frac{1}{2} \cos(\alpha + \beta)$$

$$\sin \alpha \cos \beta = \frac{1}{2} \sin(\alpha + \beta) + \frac{1}{2} \sin(\alpha - \beta)$$

$$\cos \alpha \sin \beta = \frac{1}{2} \sin(\alpha + \beta) - \frac{1}{2} \sin(\alpha - \beta)$$

Function-sum and function-difference relations

$$\sin \alpha + \sin \beta = 2 \sin \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

$$\sin \alpha - \sin \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta)$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta)$$

$$\tan \alpha + \tan \beta = \frac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta}, \quad \tan \alpha - \tan \beta = \frac{\sin(\alpha - \beta)}{\cos \alpha \cos \beta}$$

$$\cot \alpha + \cot \beta = \frac{\sin(\alpha + \beta)}{\sin \alpha \sin \beta}, \quad \cot \alpha - \cot \beta = \frac{\sin(\beta - \alpha)}{\sin \alpha \sin \beta}$$

$$\frac{\sin \alpha + \sin \beta}{\sin \alpha - \sin \beta} = \frac{\tan \frac{1}{2}(\alpha + \beta)}{\tan \frac{1}{2}(\alpha - \beta)}, \quad \frac{\sin \alpha + \sin \beta}{\cos \alpha - \cos \beta} = \cot \frac{1}{2}(\beta - \alpha)$$

$$\frac{\sin \alpha + \sin \beta}{\cos \alpha + \cos \beta} = \tan \frac{1}{2}(\alpha + \beta), \quad \frac{\sin \alpha - \sin \beta}{\cos \alpha + \cos \beta} = \tan \frac{1}{2}(\alpha - \beta)$$

Half-angle relations

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}, \quad \cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\tan \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} = \frac{1 - \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 + \cos \alpha}$$

$$\cot \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{1 - \cos \alpha}} = \frac{1 + \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 - \cos \alpha}$$

Power relations

$$\sin^2 \alpha = \frac{1}{2}(1 - \cos 2\alpha), \quad \sin^3 \alpha = \frac{1}{4}(3 \sin \alpha - \sin 3\alpha)$$

$$\sin^4 \alpha = \frac{1}{8}(3 - 4 \cos 2\alpha + \cos 4\alpha)$$

$$\cos^2 \alpha = \frac{1}{2}(1 + \cos 2\alpha), \quad \cos^3 \alpha = \frac{1}{4}(3 \cos \alpha + \cos 3\alpha)$$

$$\cos^4 \alpha = \frac{1}{8}(3 + 4 \cos 2\alpha + \cos 4\alpha)$$

$$\tan^2 \alpha = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha}, \quad \cot^2 \alpha = \frac{1 + \cos 2\alpha}{1 - \cos 2\alpha}$$

Exponential relations (α in radians), Euler's equation

$$e^{i\alpha} = \cos \alpha + i \sin \alpha, \quad i = \sqrt{-1}$$

$$\sin \alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i}, \quad \cos \alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}$$

$$\tan \alpha = -i \left(\frac{e^{i\alpha} - e^{-i\alpha}}{e^{i\alpha} + e^{-i\alpha}} \right) = -i \left(\frac{e^{2i\alpha} - 1}{e^{2i\alpha} + 1} \right)$$