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$\boxed{1}$ 2.30 :

(a) $Y_N \triangleq \text{Max}\{X_1, X_2, \ldots, X_N\}$

$\rightarrow P[Y_N \le y] = P[X_1 \le y, X_2 \le y, \ldots, X_N \le y]$

$\rightarrow F_{Y_N}(y) = [F_X(y)]^N \rightarrow \underline{\underline{p_{Y_N}(y) = N\cdot[F_X(y)]^{N-1}\cdot p_X(y)}}.$

(b) $p_X(\alpha) = \begin{cases} e^{-\alpha}, & \alpha \ge 0 \\ 0, & \alpha < 0 \end{cases} \rightarrow F_X(\alpha) = 1 - e^{-\alpha}, 0 \le \alpha < \infty.$

$$\overline{Y_N} = \int_0^\infty y \cdot N e^{-y}[1-e^{-y}]^{N-1} dy$$

$$= \sum_{j=0}^{N-1} \binom{N-1}{j} \cdot N \cdot (-1)^j \int_0^\infty y e^{-(j+1)y} dy$$

$$= \sum_{j=0}^{N-1} \binom{N-1}{j} \cdot N \cdot (-1)^j \cdot \frac{1}{(j+1)^2} = \sum_{j=0}^{N-1} \binom{N}{j+1} \frac{(-1)^j}{(j+1)}.$$

$$= \sum_{i=1}^{N} \binom{N}{i} \frac{(-1)^{i-1}}{i} \rightarrow \underline{\underline{\underset{(N=1)}{\overline{Y_N} = 1}, \ \underset{(N=2)}{\overline{Y_N} = 3/2}}}.$$

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[2] 7.4: $\overline{n(t)\,n(t-\tau)}$

2

$$= E\{[n_c(t)\sqrt{2}\cos\omega_0 t + n_s(t)\sqrt{2}\sin\omega_0 t] \cdot [n_c(t-\tau)\sqrt{2}\cos\omega_0(t-\tau) + n_s(t-\tau)\sqrt{2}\sin\omega_0(t-\tau)]\}$$

$$= R_c(\tau)[\cos\omega_0\tau + \cos(2\omega_0 t - \omega_0\tau)]$$
$$+ R_s(\tau)[\cos\omega_0\tau - \cos(2\omega_0 t - \omega_0\tau)]$$
$$+ R_{sc}(\tau)[\sin\omega_0\tau + \sin(2\omega_0 t - \omega_0\tau)]$$
$$+ R_{cs}(\tau)[-\sin\omega_0\tau + \sin(2\omega_0 t - \omega_0\tau)]$$

$$[R_c(\tau) + R_s(\tau)]\cos\omega_0\tau + [R_{sc}(\tau) - R_{cs}(\tau)]\sin\omega_0\tau$$
$$+ [R_c(\tau) - R_s(\tau)]\cos(2\omega_0 t - \omega_0\tau)$$
$$+ [R_{sc}(\tau) + R_{cs}(\tau)]\sin(2\omega_0 t - \omega_0\tau).$$

$\longrightarrow$ $n(t)$: WSS ; $\boxed{\begin{array}{l} R_c(\tau) = R_s(\tau) \\ R_{sc}(\tau) = -R_{cs}(\tau) \end{array}}$,

& $\boxed{\begin{array}{l} R_n(\tau) = 2R_c(\tau)\cos\omega_0\tau \\ \qquad + 2R_{sc}(\tau)\sin\omega_0\tau \end{array}}$.

$\boxed{3}$ **7.7:** (a) $y(t) = \begin{Bmatrix} A\,s(t) \\ -A\,s(t) \end{Bmatrix} + n(t)$,

③

$$E_s \triangleq \int_0^T s^2(t)\,dt.$$

$$\rightarrow \bar{E} = \sum_{a \in \{0,1,2\}} P[A=a] \cdot a^2 \cdot E_s = \underline{3.69\, E_s}$$

(b) $p[\text{error} \mid a] = Q\left(\sqrt{\frac{2a^2 E_s}{N_0}}\right)$

$$\rightarrow p[\text{error}] = P[A=0]\,Q(0) + P[A=1]\,Q\left(\sqrt{\frac{2E_s}{N_0}}\right) + P[A=2]\,Q\left(2\sqrt{\frac{2E_s}{N_0}}\right)$$

$$= \underline{0.005 + 0.09\,Q\left(\sqrt{\frac{2E_s}{N_0}}\right) + 0.9\,Q\left(2\sqrt{\frac{2E_s}{N_0}}\right)},$$

where $P[A=0] = 0.01$, $P[A=1] = 0.09$, $P[A=2] = 0.9$.

(c) $\frac{E_s}{N_0} \rightarrow \infty$, $p[\text{error}] = \underline{0.005}$.