II 6.1 : (a)

x_3	x_2	x_1		y_5	y_4	y_3	y_2	y_1	
0	1	1		1	0	1	0	1	(1)
1	0	1	→	0	1	0	1	0	(2)
0	1	0		0	0	1	1	0	(3)

Linear Combinations

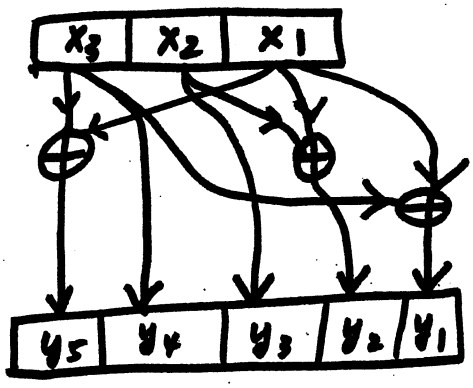
(1) & (3) : 0 0 1 → 1 0 0 1 1

(1), (2), & (3): 1 0 0 → 1 1 0 0 1

→ $\underset{\sim}{G} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}_{3\times 5}$ & $(y_5\, y_4\, y_3\, y_2\, y_1) \triangleq (x_3\, x_2\, x_1)\underset{\sim}{G}$

→
$$y_5 = x_3 + x_1$$
$$y_4 = x_3$$
$$y_3 = x_2$$
$$y_2 = x_2 + x_1$$
$$y_1 = x_3 + x_1$$

→



(b) Complete Mapping between $\underline{x}$ & $\underline{y}$

$\underline{x}$	$\underline{y}$
000	00000
001	10011
010	00110
011	10101
100	11001
101	01010
110	11111
111	01100

(2) 6.2 :

$$C: \begin{cases} \underline{y}_0 = (00000) \\ \underline{y}_1 = (01110) \\ \underline{y}_2 = (10101) \\ \underline{y}_3 = (11011) \end{cases}$$

(a) C is a linear block code.

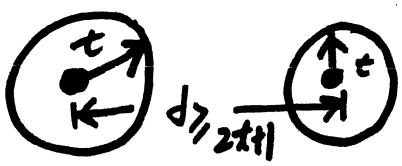
$$\underset{\sim}{G} = \begin{bmatrix} 10101 \\ 01110 \end{bmatrix}$$

$\rightarrow$ $\min w_H(C) = 3 = \min d_H(C)$, [lecture].

(b) $P_e = \sum_{i=2}^{5} \binom{5}{i} p^i (1-p)^{5-i}$, "SEC", ($d_{min} = 3 = 2\times 1 + 1$).

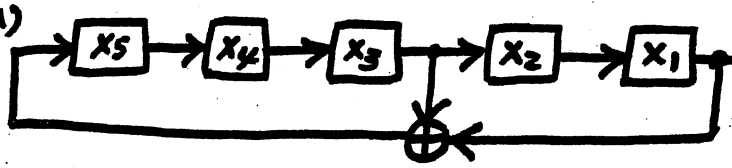
(c) For any linear block code of length n, and $d_{min} = 2t+1$,

$$P_e = \sum_{i=t+1}^{n} \binom{n}{i} p^i (1-p)^{n-i}$$ "t-EC".



No "overlapping" in "decision Regions of radius t".

[3] 6.4: (a)



(3)

Given $\underline{y}, \underline{y}' \in C_{MLSR}$

$$y_i = \begin{cases} x_i, & 1 \le i \le 5 \\ y_{i-5} + y_{i-3}, & 6 \le i \le 31. \end{cases} \quad y_i' = \begin{cases} x_i', & 1 \le i \le 5, \\ y_{i-5}' + y_{i-3}', & 6 \le i \le 31. \end{cases}$$

then $\underline{y}'' = \underline{y} + \underline{y}' \rightarrow y_i'' = y_i + y_i'$, $1 \le i \le 31$,

$$y_i'' = \begin{cases} x_i + x_i' = x_i'', & 1 \le i \le 5, \\ (y_{i-5} + y_{i-3}) + (y_{i-5}' + y_{i-3}') \\ = (y_{i-5} + y_{i-5}') + (y_{i-3} + y_{i-3}') \\ = y_{i-5}'' + y_{i-3}'', & 6 \le i \le 31. \end{cases}$$

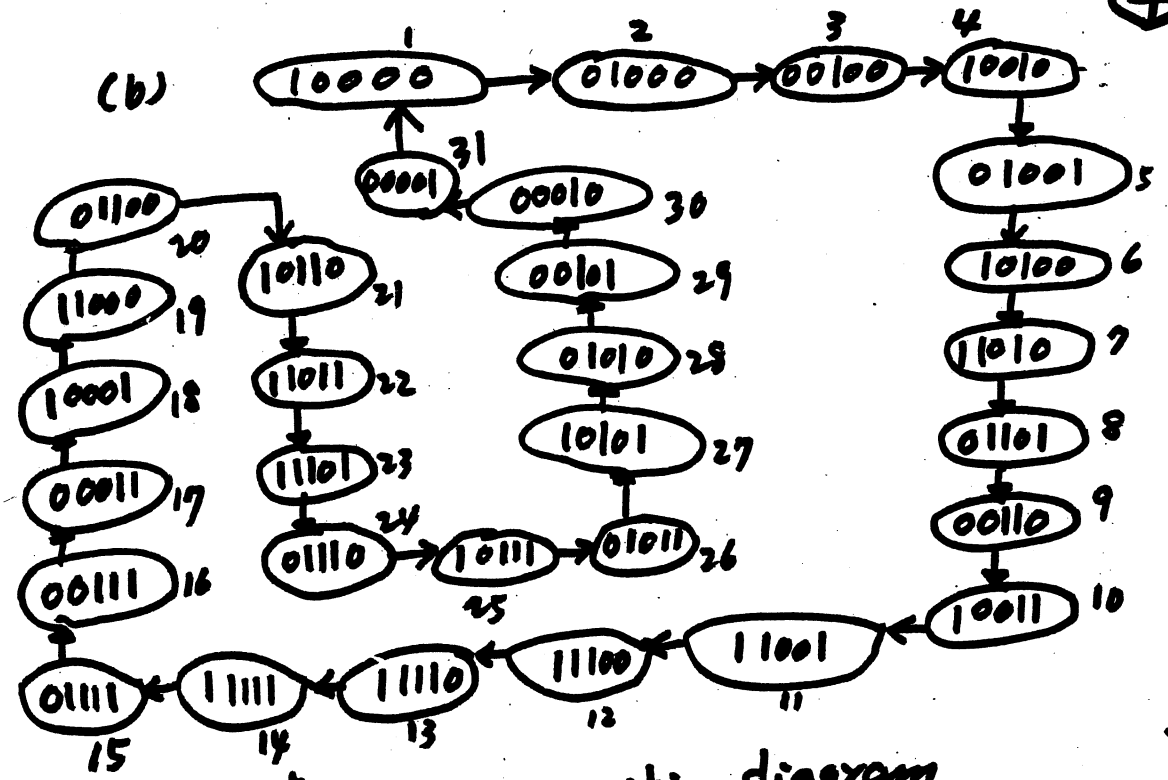
$\rightarrow \underline{y}'' \in C_{MLSR}$

$\rightarrow C_{MLSR}$ is a <u>linear</u> code.

(e) $2^K - 1$ states, $\omega_H(C_{MLSR}) = 2^{K-1}$, length $= 2^K - 1$.

$$\begin{cases} \underline{s} \cdot \underline{s}' = -E_n & (\underline{s} \ne \underline{s}') \\ \|\underline{s}\|^2 = n \cdot E_n & (n = 2^K - 1). \end{cases}$$

(b)



state transition diagram

(c) $\underline{Y} = (\underbrace{10101, 11011, 00011, 11100, 11010}_{31}, 010000)$

first ↓ Last ↓

$\rightarrow w_H(\underline{Y}) = \underline{16}$, and length $= \underline{31}$.

All nonzero codewords are "cyclic shifts" of $\underline{Y}$.

$\rightarrow w_H(C_{MLSR}) = \underline{16}$, (except "0").

$\underline{0} \xrightarrow{0 \to \sqrt{E_n}} \underline{S} = (\sqrt{E_n}, \ldots, \sqrt{E_n}) \rightarrow \sum_{i=1}^{31} s_i = 31\sqrt{E_n}$

$\underline{Y} \neq \underline{0} \xrightarrow[1 \to -\sqrt{E_n}]{0 \to \sqrt{E_n}}$ $\underline{S}$ has "16" of value "$-\sqrt{E_n}$" & "15" of value "$+\sqrt{E_n}$" $\rightarrow \sum_{i=1}^{31} s_i = -\sqrt{E_n}$

(d) $\underline{S} \cdot \underline{S}' = -16E_n + 15E_n = -E_n$

$(\underline{S} \neq \underline{S}')$

$\|\underline{S}\|^2 = 31E_n$

$\rightarrow$ Simplex.