**5.2:** (a)

$$P_e < \exp\left[-K\left(\frac{E_b}{2N_0} - \ln 2\right)\right], \quad \underline{E_{min} \triangleq (2\ln 2)N_0}$$

$$= \exp\left[-K \ln 2\left(\frac{E_b}{E_{min}} - 1\right)\right]$$

$$= 2^{-K\left(\frac{E_b}{E_{min}} - 1\right)}, \quad E_{min} \triangleq \underline{N_0 \cdot 2\ln 2}$$

$$= M^{-\left(\frac{E_b}{E_{min}} - 1\right)}, \quad M = 2^K.$$

$$P_e < 10^{-6}$$

(i) $10\log_{10}\left(\frac{E_b}{E_{min}}\right) = \underline{1\ dB} \rightarrow \underline{1.26} = \frac{E_b}{E_{min}},$

$$M^{-0.26} = 10^{-6} \rightarrow M = 10^{6/0.26} \approx \underline{\underline{10^{23}}}, \ (\underline{\underline{77\ \text{bits}}}).$$

(ii) $10\log_{10}\left(\frac{E_b}{E_{min}}\right) = \underline{3\ dB} \rightarrow \underline{2} = \frac{E_b}{E_{min}},$

$$M^{-1} = 10^{-6} \rightarrow M = \underline{\underline{10^6}}, \ (\underline{\underline{20\ \text{bits}}}).$$

(iii) $10\log_{10}\left(\frac{E_b}{E_{min}}\right) = \underline{6\ dB} \rightarrow \underline{4} = \frac{E_b}{E_{min}},$

$$M^{-3} = 10^{-6} \rightarrow M = \underline{\underline{10^2}}, \ (\underline{\underline{7\ \text{bits}}}).$$

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(b) $R = \underline{100 \text{ bps}}$

$$K = \lfloor \log_2 M \rfloor \rightarrow R = K/T \quad \text{or} \quad \boxed{T = \frac{K}{R}}$$

(i) $K = 77$ bits $\rightarrow T = \underline{\underline{770 \text{ msec}}}$

(ii) $K = 20$ bits $\rightarrow T = \underline{\underline{200 \text{ msec}}}$

(iii) $K = 7$ bits $\rightarrow T = \underline{\underline{70 \text{ msec}}}$

$$D = 3/2\, W \geq M/T \rightarrow \boxed{W \geqslant \frac{2}{3}\frac{M}{T}}$$

(i) $W \geqslant \frac{2}{3} \times \frac{10^{23}}{770 \times 10^{-3}}$

$$= \frac{2 \times 10^{23}}{3 \times 0.77} \approx \underline{\underline{10^{23} \text{ Hz}}}$$

(ii) $W \geqslant \frac{2}{3} \times \frac{10^{6}}{200 \times 10^{-3}} \approx \underline{\underline{3.3 \times 10^{6}}} \text{ Hz}$

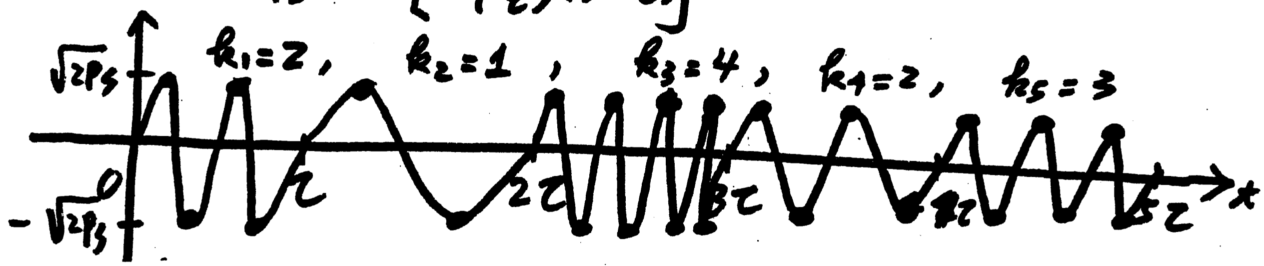
(iii) $W \geqslant \frac{2}{3} \times \frac{10^{2}}{70 \times 10^{-3}} \approx \underline{\underline{10^{3}}} \text{ Hz}$

$\boxed{2}$ 5.5: (a) $\underline{K} = (2, 1, 4, 2, 3)$

3

$$k_1 = 2, \quad \sqrt{P_s \tau} \cdot \sqrt{2/\tau}\, \sin\left[2\pi\left(\frac{2}{\tau}\right)(t-\tau)\right], \quad 0 \le t \le \tau$$

$$= \sqrt{2P_s}\, \sin\left[2\pi\left(\frac{2}{\tau}\right)(t-\tau)\right]$$




(b) All possible waveforms, and $\{\varphi_\ell(t), \varphi_k(t)\}$

$$\int_{-\tau}^{0} \sqrt{\frac{2}{\tau}} \sin\frac{2\pi \ell}{\tau} t \sqrt{\frac{2}{\tau}} \sin\frac{2\pi k}{\tau} t \, dt$$

$$= \frac{2}{2\tau}\int_{-\tau}^{0}\left[\cos\frac{2\pi(\ell-k)}{\tau}t - \cos\frac{2\pi(\ell+k)}{\tau}t\right]dt$$

$$= \frac{1}{\tau}\left\{\frac{\sin\left[\frac{2\pi(\ell-k)t}{\tau}\right]}{\left[\frac{2\pi(\ell-k)}{\tau}\right]} - \frac{\sin\left[\frac{2\pi(\ell+k)}{\tau}t\right]}{\left[\frac{2\pi(\ell+k)}{\tau}\right]}\right\}_{-\tau}^{0}$$

$$= \frac{\sin 2\pi(\ell-k)}{2\pi(\ell-k)} - \frac{\sin 2\pi(\ell+k)}{2\pi(\ell+k)} = \begin{cases} 0, & \ell \ne k. \\ 1, & \ell = k \end{cases}$$

$\rightarrow$ "orthonormal".

(c) $\underline{K} = (k_1, k_2 \cdots k_J)$, $\underline{K}' = (k_1', k_2', \ldots, k_J')$

$$\text{Prob}(\text{"}h\text{" differences}) = \binom{J}{h} \cdot \frac{(A-1)^h (1)^{J-h}}{A^J}$$

$$= \binom{J}{h}(A-1)^h / A^J, \quad 0 \le h \le J.$$

(d) $P_2[e] = P_e[\underline{s}, \underline{s}'] \triangleq Q\left(\frac{|\underline{s}-\underline{s}'|}{\sqrt{2N_0}}\right)$

$$= Q\left(\sqrt{\frac{h P_s \tau}{N_0}}\right),$$ where $|\underline{s}-\underline{s}'|^2 = |\underline{s}|^2 + |\underline{s}'|^2 - 2\underline{s}\cdot\underline{s}'$

$= P_s \cdot J \cdot \tau + P_s \cdot J \cdot \tau - 2(J-h)\cdot P_s \cdot \tau$

$= 2 \cdot h \cdot P_s \cdot \tau$, (Assume $\underline{s}$ & $\underline{s}'$ differ in $h$ components).

(e) $$\overline{P_Q(e)} = \overline{Q\left(\frac{|\underline{s}-\underline{s}'|}{\sqrt{2N_0}}\right)} = \sum_{h=0}^{J} \frac{\binom{J}{h}(A-1)^h}{A^J} Q\left(\sqrt{\frac{hP_s\tau}{N_0}}\right)$$

$$\le \sum_{h=0}^{J} \frac{\binom{J}{h}(A-1)^h}{A^J} e^{-\frac{hP_s\tau}{2N_0}}$$

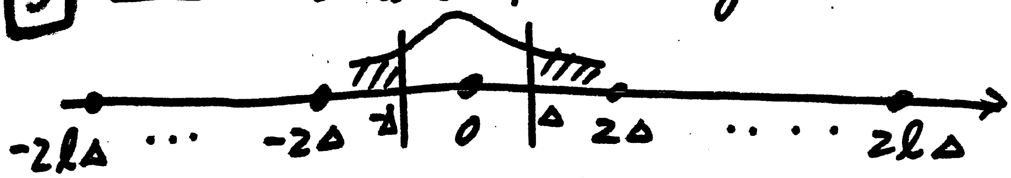
$$= \sum_{h=0}^{J} \frac{\binom{J}{h}\left[(A-1) e^{-\frac{P_s\tau}{2N_0}}\right]^h (1)^{J-h}}{A^J}$$

$$= \frac{\left[1 + (A-1) e^{-\frac{P_s\tau}{2N_0}}\right]^J}{A^J}$$

$$= \left[\frac{1 + (A-1) e^{-\frac{P_s\tau}{2N_0}}}{A}\right]^J, \quad N = A \cdot J$$

$$2^{-NR_0} = \left[\frac{1 + (A-1) e^{-\frac{P_s\tau}{2N_0}}}{A}\right]^J \rightarrow R_0 = \frac{\log_2\left[\frac{A}{1+(A-1)e^{-P_s\tau/2N_0}}\right]}{A}.$$

3 5.7 : L-level PAM signal constellation,




5

$(2\ell\Delta = A \ \& \ L = 2\ell+1)$

(a) $R_n = \log_2\left(1+\frac{A}{\Delta}\right) = \log_2(L)$

$$E_n = \frac{2}{L}\sum_{j=1}^{\ell}(2j\Delta)^2 = \frac{8\Delta^2}{L}\left(\sum_{j=1}^{\ell} j^2\right) = \frac{8\Delta^2}{L}\left[\frac{\ell(\ell+1)(2\ell+1)}{6}\right]$$

$$= \frac{8\Delta^2(\ell+1)\ell}{6} = \frac{8\Delta^2}{6}\left(\frac{L-1}{2}\right)\left(\frac{L+1}{2}\right) = \underline{\frac{\Delta^2}{3}(L^2-1)}$$

$$3E_n = \Delta^2(L^2-1) \longrightarrow \frac{3E_n}{\Delta^2}+1 = L^2$$

$$\longrightarrow \underline{L = \left(1+\frac{3E_n}{\Delta^2}\right)^{1/2}}$$

$$\rightarrow \underline{\underline{R_n = \frac{1}{2}\log_2\left(1+\frac{3E_n}{\Delta^2}\right)}} < \underline{\underline{C_n = \frac{1}{2}\log_2\left(1+\frac{2E_n}{N_0}\right)}}$$

(b) $P_e \overset{\sim}{=} 2Q\left(\frac{\Delta}{\sigma}\right) \approx e^{-\Delta^2/2\sigma^2} = e^{-\Delta^2/N_0} \quad \left(\sigma^2 = \frac{N_0}{2}\right)$

(per dimension)

$$\rightarrow \boxed{N_0 \ln\frac{1}{P_e} = \Delta^2}$$

$$\rightarrow \underline{\underline{R_n = \frac{1}{2}\log_2\left[1+\left\{\frac{3}{\ln\frac{1}{P_e}}\right\}\left(\frac{2E_n}{N_0}\right)\right]}}.$$

$$\rightarrow R_n = \frac{1}{2}\log_2\left[1+\left(\frac{3}{2\ln\frac{1}{P_e}}\right)\cdot\frac{2E_n}{N_0}\right]$$

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$$= \frac{1}{2}\log_2\left[1+D\cdot\left(\frac{2E_n}{N_0}\right)\right]$$

where $\underline{\underline{D = \dfrac{3}{2\ln\frac{1}{P_e}}}}$, and $C_n = \frac{1}{2}\log_2\left[1+\frac{2E_n}{N_0}\right]$, $\underline{\underline{D=1}}$.

| $P_e$ | $D$ (degradation factor) |
|---|---|
| $10^{-2}$ | $\frac{3}{4\ln 10} = 0.33$ |
| $10^{-4}$ | $\frac{3}{8\ln 10} = 0.16$ |
| $10^{-6}$ | $\frac{3}{12\ln 10} = 0.11$ |