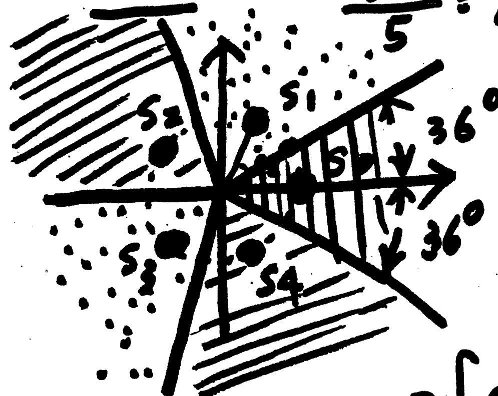
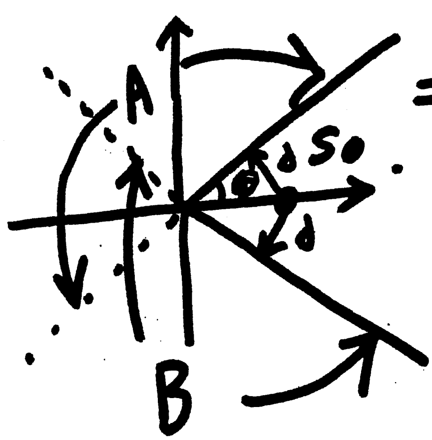
$\boxed{1}$ 4.16: (a) $\frac{360^\circ}{5} = 72^\circ$ (5-ary PSK)



(Decision Region for S_0)

(b)



$$P[\text{error} \mid S_0]$$

$$= P[A \cup B \mid S_0]$$

$$\le P[A \mid S_0] + P[B \mid S_0]$$

$$= 2Q\left(\frac{d}{\sqrt{N_0/2}}\right)$$

$$= 2Q\left(\sqrt{\frac{2E_s}{N_0}} \sin\left(\frac{\pi}{M}\right)\right) = 2p$$

$$d \triangleq \sqrt{E_s} \sin\theta$$

$$= \sqrt{E_s} \cdot \sin\left(\frac{\pi}{M}\right)$$

Similarly,

$$P[A \cup B \mid S_0] \ge P[A \mid S_0] = p$$

$$\begin{cases} \sigma = \sqrt{N_0/2} \\ \theta = \pi/M \end{cases} \rightarrow \underline{\underline{p \le P[\text{error}] \le 2p}}.$$

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$\boxed{2}$ **4.17** (a): $|\underline{\theta}_i - \underline{\theta}_j|^2 = |\underline{\theta}_i|^2 + |\underline{\theta}_j|^2 - 2\,\underline{\theta}_i \cdot \underline{\theta}_j = 2(1-\rho) \geq 0$

$\rightarrow \underline{1 \geq \rho}$.

$$\Big|\sum_{i=0}^{M-1} \underline{\theta}_i\Big|^2 = \Big(\sum_{i=0}^{M-1} \underline{\theta}_i\Big) \cdot \Big(\sum_{j=0}^{M-1} \underline{\theta}_j\Big) = \sum_{i=0}^{M-1} |\underline{\theta}_i|^2 + \sum_{0 \leq i \neq j \leq M-1} \underline{\theta}_i \cdot \underline{\theta}_j$$

$$= M \cdot 1 + M(M-1) \cdot \rho \geq 0 \rightarrow \underline{\rho \geq -\left(\tfrac{1}{M-1}\right)}, \ (M>1)$$

$$\rightarrow \underline{\underline{1 \geq \rho \geq -\left(\tfrac{1}{M-1}\right)}}, \quad (\underline{M=2}, \ |\rho| \leq 1).$$

(b)

$$\underline{s}_i = \sqrt{E_s}\, \underline{\theta}_i$$

$$\underline{a} = \frac{1}{M} \sum_{j=0}^{M-1} \underline{s}_j = \frac{\sqrt{E_s}}{M} \Big(\sum_{j=0}^{M-1} \underline{\theta}_j\Big) \rightarrow \underline{a} \cdot \underline{s}_i = \frac{E_s}{M}\left[1 + (M-1)\rho\right]$$

$$|\underline{a}|^2 = \left(\frac{\sqrt{E_s}}{M}\right)^2 \left[M + M(M-1)\rho\right] = \underline{\frac{E_s\left[1 + (M-1)\rho\right]}{M}}$$

$$(\underline{s}_i - \underline{a}) \cdot (\underline{s}_\ell - \underline{a}) = \underline{s}_i \cdot \underline{s}_\ell - \underline{a} \cdot (\underline{s}_i + \underline{s}_\ell) + |\underline{a}|^2$$

$$= \begin{cases} E_s - 2\dfrac{E_s}{M}\left[1+(M-1)\rho\right] + \dfrac{E_s}{M}\left[1+(M-1)\rho\right], & i = \ell, \\ \rho E_s \underbrace{- \dfrac{2E_s}{M}\left[1+(M-1)\rho\right] + \dfrac{E_s}{M}\left[1+(M-1)\rho\right]}_{= -\frac{E_s}{M}\left[1+(M-1)\rho\right]}, & i \neq \ell. \end{cases}$$

$$\rightarrow (\underline{s}_i - \underline{a}) \cdot (\underline{s}_\ell - \underline{a})$$

$$= \begin{cases} E_s\left(1 - \frac{1}{M}\right) - \rho E_s \left(1 - \frac{1}{M}\right) = \underline{\underline{E_s (1-\rho)\left(1 - \frac{1}{M}\right)}}, & \ell = i \\ \left[\rho - \frac{1}{M} - \left(1 - \frac{1}{M}\right)\rho\right] E_s = \underline{\underline{-\frac{1}{M}(1-\rho) E_s}}, & \ell \neq i \end{cases}$$

$$\rightarrow \begin{cases} E_s = E_g\cdot\left(1-\frac{1}{M}\right)(1-\rho) \\ \quad = \underline{E_0\cdot\left(1-\frac{1}{M}\right)}; \quad E_0 \triangleq \underline{\underline{E_g(1-\rho)}}, \quad i=l. \\ (\underline{s}_i-\underline{a})\cdot(\underline{s}_l-\underline{a}) = \underline{\underline{-\frac{E_0}{M}}}, \quad i\neq l. \end{cases}$$

$\boxed{3}$ 4.18:

(a) Set a $\rightarrow$ $\int_0^4 s_1^2(t)\,dt = \int_0^4 s_2^2(t)\,dt = \int_0^4 s_3^2(t)\,dt$

$= \int_0^4 s_4^2(t)\,dt = \underline{\underline{E_s}}$

Set b $\rightarrow$ $\int_0^4 s_1^2(t)\,dt = \int_0^4 s_2^2(t)\,dt = \int_0^4 s_3^2(t)\,dt$

$= \int_0^4 s_4^2(t)\,dt = \underline{\underline{E_s}}$

(b) Set a $\rightarrow$ $\underline{s}_1 = \left(\sqrt{\frac{E_s}{2}}, 0, 0, \sqrt{\frac{E_s}{2}}\right)$

$\underline{s}_2 = \left(0, \sqrt{\frac{E_s}{2}}, \sqrt{\frac{E_s}{2}}, 0\right)$

$\underline{s}_3 = \left(\sqrt{\frac{E_s}{2}}, 0, \sqrt{\frac{E_s}{2}}, 0\right)$

$\underline{s}_4 = \left(0, \sqrt{\frac{E_s}{2}}, 0, \sqrt{\frac{E_s}{2}}\right)$

d_E^2	$\underline{s}_1$	$\underline{s}_2$	$\underline{s}_3$	$\underline{s}_4$
$\underline{s}_1$		$2E_s$	E_s	E_s
$\underline{s}_2$			E_s	E_s
$\underline{s}_3$				$2E_s$
$\underline{s}_4$				

$\rightarrow \underline{\underline{d_{min} = \sqrt{E_s}}}$

$\rightarrow P_e \le \underline{\underline{3\cdot Q\left(\sqrt{\frac{E_s}{2N_0}}\right)}}$

(Union Bound)

Set b → $\underline{s}_1 = (0, 0, 0, \sqrt{E_s})$

$\underline{s}_2 = (0, \sqrt{E_s}, 0, 0)$

$\underline{s}_3 = (0, 0, \sqrt{E_s}, 0)$

$\underline{s}_4 = (\sqrt{E_s}, 0, 0, 0)$

d_E^2	$\underline{s}_1$	$\underline{s}_2$	$\underline{s}_3$	$\underline{s}_4$
$\underline{s}_1$		$2E_s$	$2E_s$	$2E_s$
$\underline{s}_2$			$2E_s$	$2E_s$
$\underline{s}_3$				$2E_s$
$\underline{s}_4$				

$\rightarrow d_{min} = \sqrt{2E_s}$

$\rightarrow P_e \leq 3 \cdot Q\left(\sqrt{\frac{E_s}{N_0}}\right)$

⇒ Set b is 3dB ($10\log_{10}(2)$) more effective than set a.