Chang, Shih-Chun "Homework Solutions No. 01". Lecture Notes. ECE 630 – Section 001. Spring 2011. 1-6

HWO11012611.pdf

**1** **2.4:**

$P[B] = 1/2$, $P[C] = 1/4$, $P[D] = 1/4$.

| URN A | URN B | URN C | URN D |
|---|---|---|---|
| 6B, 3C, 3D | 5R, 5W | 4R, 6W | 2R, 8W |

a. Sample Space: $\{(B,R), (B,W), (C,R), (C,W), (D,R), (D,W)\}$, (pair of outcomes).

b. $P[B|R] = P[B]\,P[R|B] / P[R]$

$$= \frac{1/2 \times 1/2}{1/2 \times 1/2 + 1/4 \times 4/10 + 1/4 \times 2/10} = \underline{\underline{\frac{5}{8}}},$$

(where $P[R] = P[B]P[R/B] + P[C]P[R/C] + P[D]P[R/D]$,
$\rightarrow P[R] = \underline{2/5}$.)

c.
$P[C,R] = P[C]\,P[R|C] = \frac{1}{4} \times \frac{4}{10} = \underline{\underline{\frac{1}{10}}}$
$P[C]\,P[R] = \frac{1}{4} \times \frac{2}{5} = \underline{\underline{\frac{1}{10}}}$
$\rightarrow (C,R)$ \underline{independent}

d. $P[R] = \underline{2/5} \rightarrow P[W] = \underline{3/5}$. (why?)

$P[R|B] = \underline{1/2} \neq P[R] \rightarrow$ dependent, $(B,R)$.
$P[R|C] = \underline{2/5} = P(R) \rightarrow$ <u>independent</u>, $(C,R)$.
$P[R|D] = \underline{1/5} \neq P[R] \rightarrow$ dependent, $(D,R)$.

$P[W|B] = 1/2 \neq P[W] \rightarrow$ dependent, $(B,W)$
$P[W/C] = 3/5 = P[W] \rightarrow$ <u>independent</u>, $(C,W)$
$P[W/D] = 4/5 \neq P[W] \rightarrow$ dependent, $(D,W)$.

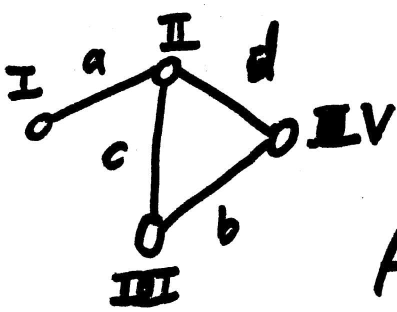
(why?) ➡ "NO"

$\boxed{2}$ 2.8 (a) $\Omega = \{\omega_0, \dots, \omega_{15}\}$ (2)

| outcome \ link | a | b | c | d |
|---|---|---|---|---|
| $\omega_0$ | off | off | off | off |
| $\omega_1$ | off | off | off | on |
| $\vdots$ | | $\vdots$ | | |
| $\omega_{15}$ | on | on | on | on |

← (state of links)

(b) $A = \{\omega : \text{I and IV can communicate}\}$




| | a | b | c | d |
|---|---|---|---|---|
| A: | 1 | 0 | 0 | 1 |
| | 1 | 0 | 1 | 1 |
| | 1 | 1 | 0 | 1 |
| | 1 | 1 | 1 | 0 |
| | 1 | 1 | 1 | 1 |

ON : 1
OFF : 0

$\begin{cases} P[ON] = p \\ P[OFF] = 1-p \end{cases}$

$$P[A] = p^2(1-p)^2 + 3p^3(1-p) + p^4$$
$$= \underline{\underline{p^2 + p^3 - p^4}}.$$

(c) $B = \{\omega : \text{II and III can communicate}\}$

| | a | b | c | d |
|---|---|---|---|---|
| B | x | 0 | 1 | 0 |
| | x | 0 | 1 | 1 |
| | x | 1 | 1 | 0 |
| | x | 1 | 0 | 1 |
| | x | 1 | 1 | 1 |

$$P[B] = p(1-p)^2 + 3p^2(1-p) + p^3 = \underline{\underline{p + p^2 - p^3}}.$$

(x: irrevalent) → x can be '0' or '1'.

③

(d) $AB \triangleq A \cap B$

$= \{ (1111), (1101), (1110), (1011) \}$

$P[AB] = p^4 + 3p^3(1-p)$

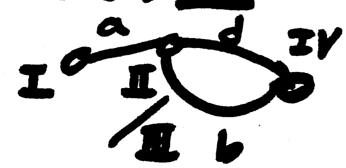
$= p^4 + 3p^3 - 3p^4 = \underline{\underline{3p^3 - 2p^4}}.$

(e) $P[A] = P[A \mid c:ON]\, P[c:ON]$

$\quad\;\; + P[A \mid c:OFF]\, P[c:OFF]$

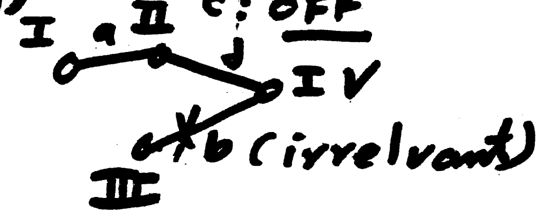
(e)

(i) c: ON




$I \leftrightarrow IV: [1-(1-p)^2] \cdot p$

(ii) c: OFF




$= p^2 \cdot [1-(1-p)^2]$

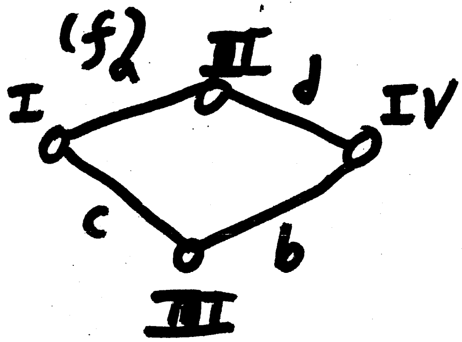
$+ (1-p)[\,p^2\,]$

$= p^2 [1 - (1-p)^2 + (1-p)]$

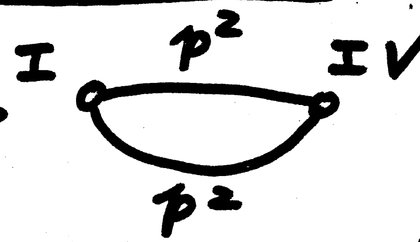
$= p^2 [1 - 1 + 2p - p^2 + 1 - p]$

$= p^2 [1 + p - p^2] = \underline{\underline{p^2 + p^3 - p^4}}$, ("same" as (b)).

(f)




$\rightarrow$




$\rightarrow P[A]_{(f)} = 1 - (1-p^2)^2$

$= 1 - 1 + 2p^2 - p^4$

$= \underline{\underline{2p^2 - p^4}}$

$P[A]_{(f)} - P[A]_{(e)}$

$= p^2 - p^3 = \underline{\underline{p(1-p) > 0}}$, (increase!)

$\boxed{3}$ <u>2.11</u>: (i) $P[a]=0.3,\ P[b]=0.5,\ P[c]=0.2,$ ④

$$\begin{cases} P[1|a]P[a]=\underline{0.18}, & P[1|b]P[b]=0.05, & P[1|c]P[c]=0.02 \\ P[2|a]P[a]=0.09, & P[2|b]P[b]=\underline{0.25}, & P[2|c]P[c]=0.02 \\ P[3|a]P[a]=0.03, & P[3|b]P[b]=\underline{0.2}, & P[3|c]P[c]=0.16 \end{cases}$$

$\rightarrow D_a=\{1\},\ D_b=\{2,3\},\ D_c=\phi.$

(ii)
$$P[\text{correct}]=P[a]P[1|a]+P[b]P[2|b]+P[b]P[3|b]$$
$$=0.18+0.25+0.2=\underline{0.63}$$

$\rightarrow P[\text{error}]=\underline{0.37}.$

(iii) Always choose "b", $P[\text{error}]=\underline{0.5}.$

$\boxed{4}$ <u>2.12</u>: (a)
$$\begin{cases} P[a|0]P[0]=\underline{0.49}, & P[a|1]P[1]=0.09, \\ P[b|0]P[0]=\underline{0.14}, & P[b|1]P[1]=0.06, \\ P[c|0]P[0]=0.07, & P[c|1]P[1]=\underline{0.15} \end{cases}$$

$\rightarrow D_0=\{a,b\},\ D_1=\{c\}$

$\rightarrow P[\text{correct}]=0.49+0.14+0.15=\underline{0.78}$

$\rightarrow P[\text{error}]=\underline{0.22}.$

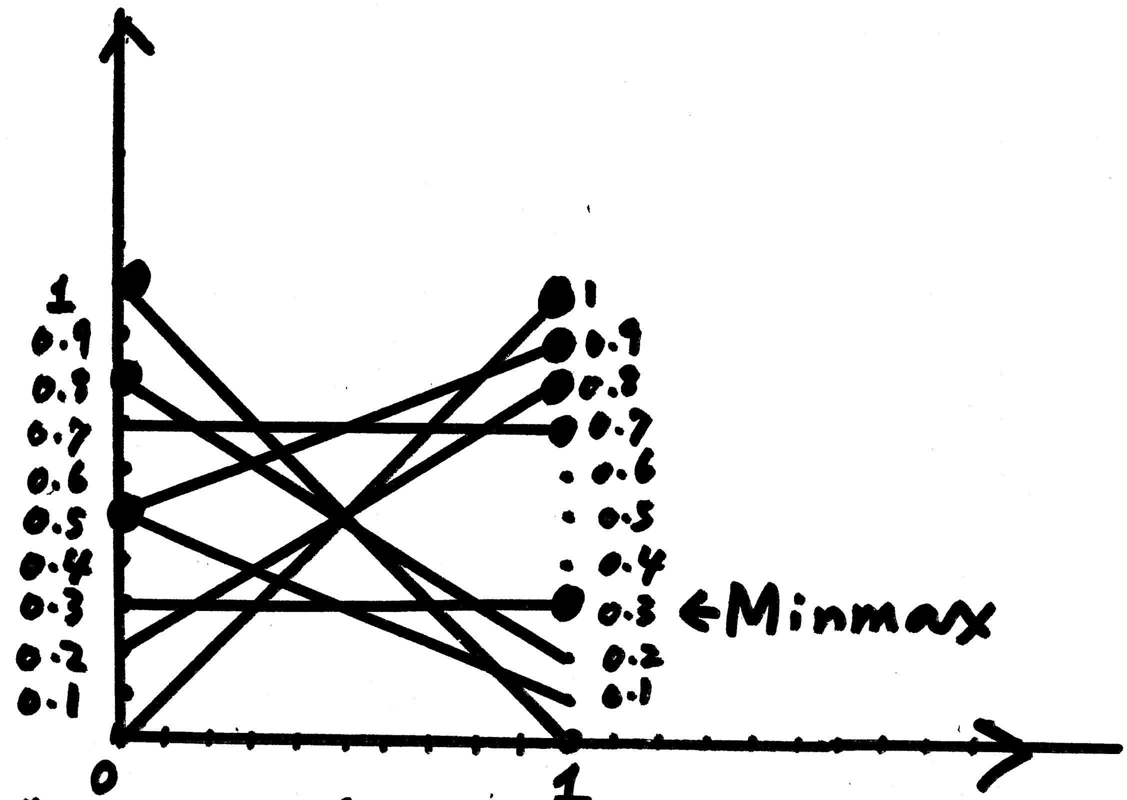
(b)

| Rule \ outcomes | a | b | c | P[error] |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 − P(0) |
| 2 | 0 | 0 | 1 | 0.5 − 0.4 P(0) |
| 3 | 0 | 1 | 0 | 0.8 − 0.6 P(0) |
| 4 | 0 | 1 | 1 | 0.3 |
| 5 | 1 | 0 | 0 | 0.7 |
| 6 | 1 | 0 | 1 | 0.2 + 0.6 P(0) |
| 7 | 1 | 1 | 0 | 0.5 + 0.4 P(0) |
| 8 | 1 | 1 | 1 | P(0) |

5

(Decision Table)

(c)




**Rule# 4** : $D_0 = \{a\}$, $D_1 = \{b, c\}$

$\longrightarrow P[\text{error}] = \underline{\underline{0.3}}$ $(< 0.5 < 0.7 < 0.8 < 0.9 < 1)$

⑤ 2.36: [a (i) ~ (v) only]  ⑥

a. (i) (True) $\overline{y} = \sum_{i=1}^{n} \overline{x_i}$, (why?)

(ii) (False) $\overline{y^2} = \sum_{i=1}^{n} \overline{x_i^2} + \sum_{i \neq j}\sum \overline{x_i} \cdot \overline{x_j}$

$\neq \sum_{i=1}^{n} \overline{x_i^2}$, (if $\overline{x_i} \neq 0$, $1 \le i \le n$).

(iii) (False) $\overline{y^3} \neq \sum_{i=1}^{n} \overline{x_i^3}$, (if $\overline{x_i} \neq 0$, $\overline{x_i^2} \neq 0$, $1 \le i \le n$).

(iv) (False)
$$M_y(v) \triangleq E\left[e^{jvy}\right]$$
$$= E\left[e^{jv\left(\sum_{i=1}^{n} x_i\right)}\right]$$
$$= E\left[e^{jvx_1}\right] E\left[e^{jvx_2}\right] \cdots E\left[e^{jvx_n}\right]$$
$$= \prod_{i=1}^{n} M_{x_i}(v) \neq \sum_{i=1}^{n} M_{x_i}(v).$$

(v) (True)
$$\sigma_y^2 \triangleq E\left[\left(\sum_i x_i\right)^2\right] - \left[\sum_i E(x_i)\right]^2$$
$$= \sum_i E(x_i^2) + \sum_{i \neq j}\sum E(x_i)E(x_j)$$
$$- \left[\sum_i E(x_i)^2 + \sum_{i \neq j}\sum E(x_i)E(x_j)\right]$$
$$= \sum_i \left[E(x_i^2) - E(x_i)^2\right] = \sum_i \sigma_{x_i}^2.$$